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PRESSING OF A COMPACT PLASTIC MATERIAL

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Pressing in a closed mold has been considered by many authors [1-5]. In contrast to others, it is shown in the present work that the compaction process occurs in two stages: in the first deformation it is only in the region adjacent to the piston, and in the second it is in the whole volume of the material. In the first stage around the bottom of the mold there is a rigid (undeformed) zone. The position of the boundary between the rigid and deforming zones depends on the amount of upsetting. The first stage ends when this boundary reaches the bottom of the mold. Presence of a densification front is confirmed by experiment [6].

With relatively low density at the rubbing surfaces the Coulomb friction rule operates. With an increase in density normal pressure and frictional force grow in an unlimited way and at a certain instant reach a maximum value permissible by the flow condition. Then the Coulomb friction rule is not valid and the Prandtl friction rule takes effect. The existence of two friction zones, i.e., Coulomb and Prandtl, at the rubbing surfaces is possible at a certain stage of the process. With a further increase in density the Coulomb zone disappears and the Prandtl rule operates on the whole surface of the mold.

Statement of the Problem. We consider pressing of an axisymmetrical sleeve with an internal rod. We introduce a cylindrical coordinate system (r, θ, z), axis z of which coincides with the axis of symmetry of the pressed article [Fig. 1: 1) mold; 2) piston; 3) rod].

The radial velocity of particles v_r should revert to zero at the surface of the rod and the mold wall, i.e., this value is small. We assume that $v_r = 0$. The corresponding equilibrium equation of obtained by the Hill method [7]. The equation of virtual powers has the form

$$\int\limits_{R_1}^{R_2}\int\limits_{0}^{h}\left(\sigma_z\frac{\partial v_z}{\partial z}+\tau_{rz}\frac{\partial v_z}{\partial r}\right)r\,dr\,dz=\int\limits_{R_1}^{R_2}\sigma_z v_z r\,dr\Big|_{z=h}+\int\limits_{0}^{h}\tau_{rz}v_z r\,dz\Big|_{r=R_1}^{r=R_2}. \qquad (1)$$

Here R_2, R_1 are mold and rod radii; h is current blank height; v_z is projection of velocity on axis z; σ_z, τ_{rz} are normal and tangential stress tensor components.

In order to satisfy boundary conditions at the bottom of the container and base of the piston we assume that v_z does not depend on r, and we perform in the left-hand part of Eq. (1) integration with respect to parts:

$$\int\limits_{0}^{h}\left[\int\limits_{R_1}^{R_2}\left(r\frac{\partial\sigma_z}{\partial z}+\frac{\partial(r\tau_{rz})}{\partial r}\right)dr\right]v_z\,dz=0. \qquad (2)$$

Since v_z is a derivative of function z, then from (2) it follows that the expression in square brackets should be equal to zero. The equilibrium equation is written as

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$\partial S/\partial z + T_1 R_1 + T_2 R_2 = 0$ $\left(S = \int_{R_1}^{R_2} r\sigma_z\, dr\right.$ is proportional to the average value of σ_z over the cross section; T_1, T_2 are specific forces of friction at the rod surface and the mold walls).

We assume that the effect of the force of external friction appears in narrow zones adjacent to the side surfaces of the pressed volume, and that the distribution of σ_z over the cross section is close to uniform. Then $S = 0.5(R_2^2 - R_1^2)\sigma_z$ and the equilibrium equation takes the form

$$\partial\sigma_z/\partial z + T_1 2R_1/(R_2^2 - R_1^2) + T_2 2R_2/(R_2^2 - R_1^2) = 0. \tag{3}$$

Values of T_1 and T_2 are determined by the friction rule. Thus, static boundary conditions are considered in the equilibrium equations. In the future we assume that $T_1 = T_2 = T$. Equation (3) may be obtained by the plane section method. However, use of the Hill method connects this equation with the suggested kinematic character given previously.

We assume that the material obeys the Green flow condition

$$\sigma^2/\alpha + \tau^2/\beta = k^2. \tag{4}$$

Here σ is average stress; τ is tangential stress intensity; k is the flow limit in shear for the base material; α, β are known functions of ρ (ρ is relative density equal to the ratio of dimensional density to solid phase density). From equations of the associated flow rule with the assumptions adopted with respect to stresses and velocities the stressed state for the process in question is represented by the equations [5]

$$\sigma_r = \sigma_\theta = -\frac{(3\alpha - 2\beta)}{\sqrt{3}\,(3\alpha + 4\beta)^{1/2}}k, \quad \sigma_z = -\frac{1}{\sqrt{3}}(3\alpha + 4\beta)^{1/2}k \tag{5}$$

(σ_r, σ_θ, σ_z are principal normal stresses). By substituting the last of Eqs. (5) in (3) we have an equation determining the dependence of density on coordinate z:

$$\partial\rho/\partial z = 1/g(\rho), \; g(\rho) = k(R_2 - R_1)/[4\sqrt{3}T(3\alpha + 4\beta)^{1/2}](3d\alpha/d\rho + 4d\beta/d\rho) \tag{6}$$

(T is a known function of density with any friction rule). With the Coulomb friction rule $T = f|\sigma_r|$ [σ_r is from (5)]. In order to find v_z we have a continuity equation which with the assumptions made is written as

$$\partial\rho/\partial t + v_z\partial\rho/\partial z + \rho\partial v_z/\partial z = 0. \tag{7}$$

Relationships (6) and (7) together with boundary and starting conditions determine the dependence of ρ and v_z on x and t.

First Stage of Compaction. We assume that in the initial instant density is uniformly distributed ($\rho = \rho_0$) and the height of the blank is h_0. Solution of Eq. (6) takes the form

$$z + \varphi(t) = G(\rho), \quad G(\rho) = \int_{\rho_0}^{\rho} g(\rho)\, d\rho \tag{8}$$

[$\varphi(t)$ is derivative function of time]. Relationship (8) is rewritten in the form $\rho = F(z + \varphi(t))$ (F is a function inverse to G). It can be seen that the change in ρ with time has a wavy character. If $\varphi(t)$ is an increasing function, then the wave moves in the negative direction of axis z. Distribution of density at the current instant of time t is presented as: $\rho = \rho_0$ with $h_A(t) \geqslant z \geqslant 0$ (rigid zone), $\rho = F(z + \varphi(t))$ with $h(t) \geqslant z \geqslant h_A(t)$ (compaction zone), where h(t) is current height of the working volume, $h_A(t)$ is the height corresponding to the boundary of the deforming and rigid zones which is determined from the condition of continuity of density at the boundary of these zones ($\rho = \rho_0$ with $z = h_A$). From (8) it follows that $h_A = -\varphi(t)$. It is required that in the initial instant $h_A = h_0$, i.e., that there is no compaction zone. Then function $\varphi(t)$ should satisfy the condition $\varphi(0) = -h_0$.

The process of compaction may be presented as follows. In the initial instant density over the whole length of the blank is distributed uniformly. Then a compaction zone develops under the piston in which $\rho = F(z + \varphi(t))$. It occupies section $h_A(t) \leqslant z \leqslant h(t)$, and it propagates downwards with velocity $|\dot{h}_A| = \dot{\varphi}$ ($\dot{h}_A = dh_A/dt$, $\dot{\varphi} = d\varphi/dt$). With $0 \leqslant z < h_A$ deformation is absent and the density retains its initial value.

In order to determine $\varphi(t)$ and the projection of velocity v_z we consider continuity Eq. (7). By using (8) we bring it to the form $\rho\partial v_z/\partial\rho + v_z + \dot{\varphi} = 0$, whence $v_z = -\dot{\varphi}(t) + \psi(t)/\rho$

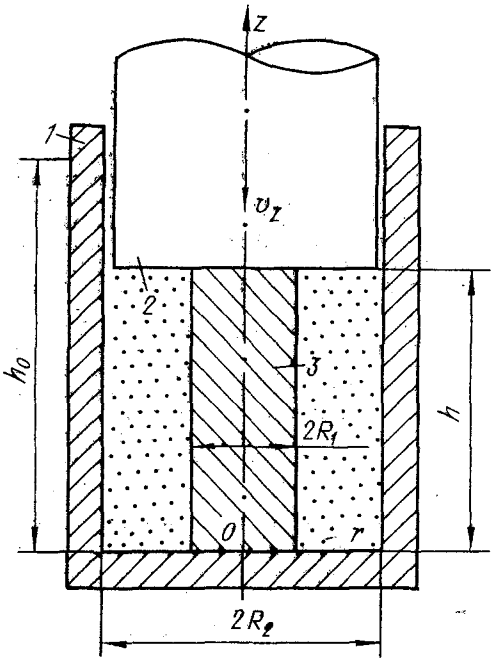

Fig. 1

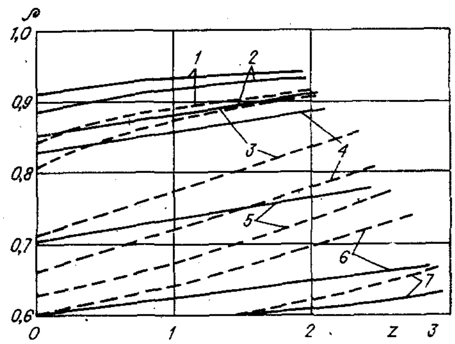

Fig. 2

[$\psi(t)$ is derivative function for time].

In order to find φ and ψ we have two boundary conditions: the first expresses the continuity condition for density and velocity at the boundary of the deforming and rigid zones: $v_z = 0$ with $\rho = \rho_0$, and the second expresses the condition of impenetrability of the bottom of the piston: $v_z = \dot{h}$ with $z = h$. From the first condition it follows that $\psi(t) = \dot{\varphi}(t)\rho_0$, and the second with the relationship $v_z = -\dot{\varphi}(t) + \psi(t)/\rho$ leads to a differential equation for $\gamma(t)$:

$$d\varphi = [\rho_m/(\rho_0 - \rho_m)]dh \tag{9}$$

[$\rho_m = F(h + \varphi)$ is density under the piston]. Equation (9) gives the dependence of φ on h, from the initial condition it follows that $\varphi(h_0) = -h_0$, and this determines the dependence of φ on t since h(t) is known. By using the relationship $h + \varphi(t) = G(\rho_m)$, it is possible to consider φ as a function of ρ. We differentiate this equality and substitute the expression obtained in (9), which after integrating we have

$$\varphi = \frac{1}{\rho_0}\int_{\rho_0}^{\rho_m} \rho_m g(\rho_m)\, d\rho_m - h_0. \tag{10}$$

The distribution of density over the height is obtained from (8). By means of (9) we find that

$$v_z = \frac{\rho_m}{\rho}\frac{(\rho_0 - \rho)}{(\rho_0 - \rho_m)}\dot{h}.$$

<u>Second Stage of Compaction.</u> Compaction occurs throughout the whole volume of pressed material. Instant of time t_* and the height of the working section h_* corresponding to the start of the second stage are determined from the condition $h_A = 0$. Since $h_A = -\varphi(t)$, then t_* is found from the equation $\varphi(t_*) = 0$. Since h is a prescribed function of time, then h_* is also known. Density distribution in the second stage as before is described by the function $\rho = F(z + \varphi(t))$. However, instead of boundary condition $v_z = 0$ with $\rho = \rho_0$ it should be assumed that $v_z = 0$ with $z = 0$. Since $v_z = -\dot{\varphi} + \psi/\rho$, then this condition leads to the relationship $\psi = \dot{\varphi}\rho_n$, where ρ_n is density at the bottom of the mold (z = 0), then

$$v_z = \dot{\varphi}(\rho_n/\rho - 1). \tag{11}$$

From the second boundary condition ($v_z = \dot{h}$ with $z = h$) we have

$$(\dot{h} + \dot{\varphi})\rho_m = \dot{\varphi}\rho_n. \tag{12}$$

Since $h + \varphi = G(\rho_m)$, then $\dot{h} + \dot{\varphi} = g(\rho_m)\dot{\rho}_m$. In addition, $\varphi = G(\rho_n)$, so that $\dot{\varphi} = g(\rho_n)\dot{\rho}_n$. By substituting these conditions in (12) we obtain $\rho_n g(\rho_n) d\rho_n = \rho_m g(\rho_m) d\rho_m$. Whence

$$\int_{\rho_0}^{\rho_n} \rho g(\rho)\, d\rho = \int_{\rho_0}^{\rho_m} \rho g(\rho)\, d\rho - c \quad (c = \text{const}).$$

Constant c is found from the condition $\rho_m = \rho_m^* = \rho_m(t_*)$, $\rho_n = \rho_0$ at the instant of the start of the second stage. Finally we obtain the relationship between current values of ρ_m and ρ_n:

$$\int_{\rho_0}^{\rho_n} \rho g(\rho)\, d\rho = \int_{\rho_m^*}^{\rho_m} \rho g(\rho)\, d\rho, \tag{13}$$

and it determines ρ_n as a function of ρ_m. Since $\varphi = G(\rho_n)$, then $\varphi = \varphi(\rho_m)$. Thus, all of the parameters of the process are expressed in terms of ρ_m. Since ρ_m is clearly a factor of h, then the solution does not depend implicitly on time, but it depends on h and z as it should in a plastic material.

In order to obtain the final equation for velocity in the second stage we express $\dot{\varphi}$ in terms of h by means of (12) and we substitute it in (11):

$$v_z = \frac{(\rho_n - \rho)}{(\rho_n - \rho_m)} \frac{\rho_m}{\rho} \dot{h}. \tag{14}$$

The solution built up above depends on the form of functions α and β in the flow condition (4) and the rule for friction at the mold and rod walls. For α and β we take [8]:

$$\alpha = (4/3)\rho^4/(1-\rho), \quad \beta = \rho^3. \tag{15}$$

The friction rule for the surfaces in contact depends on the stressed state and the Coulomb friction coefficient f which is assumed to be constant. If the stressed state is such that specific forces of friction found by the Coulomb rule are less than τ_{max} (τ_{max} is the maximum tangential stress permitted by the flow condition with a given value of average stress σ), then $T = f|\sigma_r| \leqslant \tau_{max}$, and if this inequality is not fulfilled than the Prandtl friction rule $T = \tau_{max}$ should be adopted.

Since stresses determined by (5) satisfy the flow condition and they are the principal stresses, then $\tau_{max} = |\sigma_r - \sigma_\theta|/2$ and consequently taking account of (15) we have

$$\tau_{max} = (\sqrt{3}/2)\rho^{3/2}(1-\rho)^{1/2}. \tag{16}$$

In the initial stage while ρ and $|\sigma_r|$ are small the Coulomb friction rule is fulfilled. It follows from (5) that $|\sigma_r| \to \infty$ with $\rho \to 1$, therefore during compaction specific friction forces increase and in a certain instant of time they reach τ_{max}. This occurs first at those points of the friction surface which are adjacent to the piston since maximum density occurs there. Further upsetting leads to spreading of the Prandtl friction zone over the whole height of the seat of deformation.

We determine conditions for development of a Prandtl friction zone and disappearance of the Coulomb friction zone. For this we compare τ_{max} from (16) with values of specific friction forces with the Coulomb rule ($T = f|\sigma_r|$, σ_r is given by relationship (5). As a result of this we obtain

$$\rho_1 = (3 + 2f)/3(2f + 1).$$

The condition $\rho_m = \rho_1$ corresponds to development of a Prandtl friction zone, and $\rho_n = \rho_1$ corresponds to disappearance of a Coulomb friction zone. If f = 0.1 and 0.2, then ρ_1 = 0.89 and 0.81. In the third stage the density is low and as a rule friction forces do not reach τ_{max} and this case is not studied.

The Case when a Coulomb Friction Rule Operates over the Whole Height of the Seat of Deformation. We consider the initial stage of compaction ($\rho_m \leqslant \rho_1$). Here over the whole extent of the seat of deformation the Coulomb friction rule operates, i.e., $T = f|\sigma_r| = f\frac{k}{\sqrt{3}} \times \frac{\rho^{3/2}(3\rho - 1)}{(1-\rho)^{1/2}}$. According to (6) and (15) we have

$$g(\rho) = \frac{(R_2 - R_1)}{2f} \frac{(3 - 2\rho)}{(1-\rho)\rho(3\rho - 1)},$$
$$G(\rho) = \frac{(R_2 - R_1)}{2f} \ln\left[c_0 \frac{(3\rho - 1)^{3,5}}{\rho^3(1-\rho)^{0,5}}\right] \quad (c_0 = \rho_0^3(1-\rho_0)^{0,5}(3\rho_0 - 1)^{-3,5}). \tag{17}$$

In the first stage of compaction function φ is determined by relationship (10):

$$\varphi = \frac{(R_2 - R_1)}{2f\rho_0} \ln\left\{\left(\frac{3\rho_m - 1}{3\rho_0 - 1}\right)^{7/6}\left(\frac{1-\rho_0}{1-\rho_m}\right)^{1/2}\right\} - h_0. \tag{18}$$

The current height of the working volume

$$h = G(\rho_m) - \varphi(\rho_m). \tag{19}$$

Knowing relationship (18), by means of (8) and (17) it is possible to calculate the distribution of density and pressing pressure for different values of ρ_m. In this way (19) determines the position of the piston at a given instant of time.

Results of calculation with $R_1 = 0$, $h_0/R_2 = 3$, $\rho_0 = 0.6$, $f = 0.1$ (solid lines) and 0.2 (broken lines) are given in Fig. 2 (curves 6 and 7).

The value of ρ_m corresponding to the end of the first compaction stage ρ_m^* is found from the equation $\varphi = 0$: $\rho_m^* = 0.614$ with $f = 0.2$, $\rho_m^* = 0.607$ with $f = 0.1$. In the second compaction stage φ is determined by the equation $\varphi = G(\rho_n)$. The value of ρ_n is obtained from (13) which now takes the form

$$\left(\frac{3\rho_n - 1}{3\rho_0 - 1}\right)^{7/6}\left(\frac{1-\rho_0}{1-\rho_n}\right)^{1/2} = \left(\frac{3\rho_m - 1}{3\rho_m^* - 1}\right)^{7/6}\left(\frac{1-\rho_m^*}{1-\rho_m}\right)^{1/2}. \tag{20}$$

Results of calculating the density distribution are presented in Fig. 2 (curves 4 and 5). This solution is in force while $\rho_m \leqslant \rho_1$. If $\rho_1 < \rho_m$, then it is necessary to consider the combined existence of Coulomb and Prandtl friction zones.

<u>The Case when there are Both Zones.</u> This period lasts while the inequality $\rho_m \geqslant \rho_1 \geqslant \rho_n$. is fulfilled. Each zone corresponds to its solution:

for the Coulomb friction zone

$$z + \varphi_c(t) = G_c(\rho),\ G_c(\rho) = \int_{\rho_0}^{\rho} g_c(\rho)\, d\rho,\quad \rho = F_c(z + \varphi_c),\ v_c = -\dot{\varphi}_c + \psi_c/\rho;$$

for the Prandtl friction zone

$$z + \varphi_p(t) = G_p(\rho),\ G_p(\rho) = \int_{\rho_0}^{\rho} g_p(\rho)\, d\rho,\quad \rho = F_p(z + \varphi_p),$$
$$v_p = -\dot{\varphi}_p + \psi_p/\rho. \tag{21}$$

Here $g_c(\rho)$ and $G_c(\rho)$ are determined from (17), and

$$g_p(\rho) = \frac{(R_2 - R_1)}{4}\frac{(3 - 2\rho)}{\rho(1-\rho)^2},\quad G_p(\rho) = \frac{(R_2 - R_1)}{4}\left\{\left[3\ln\left(\frac{\rho}{1-\rho}\right) + \left(\frac{1}{1-\rho}\right)\right] - \left[3\ln\left(\frac{\rho_0}{1-\rho_0}\right) + \left(\frac{1}{1-\rho_0}\right)\right]\right\}. \tag{22}$$

The boundary of zone z^* is found from the condition $\rho = \rho_1$. Whence $z^* = G_c(\rho_1) - \varphi_c(t)$ and $z^* = G_p(\rho_1) - \varphi_p(t)$, and consequently $\varphi_p(t) = \varphi_c(t) - G_c(\rho_1) + G_p(\rho_1)$ and $\dot{\varphi}(t) = \dot{\varphi}_c(t)$. Since at the boundary of the zones the projection of velocity v is continuous, then $\psi_c(t) = \psi_p(t)$. The Coulomb friction zone is adjacent to the bottom of the mold, and therefore $v_c = 0$ with $z = 0$, then $\dot{\varphi}_c(t)\rho_p = \psi_c(t)$. The Prandtl friction zone is adjacent to the piston, and this means $\varphi_c(t) + \psi_c(t)/\rho_m = \dot{h}$. By combining the last two equalities we obtain

$$\dot{\varphi}_c(t)(\rho_n/\rho_m - 1) = \dot{h}. \tag{23}$$

By determining from $h = G_p(\rho_m) - \varphi_p(t)$ the value of $\dot{h}$ and substituting it taking account of $\varphi_p(t) = \varphi_c(t)$ in (23) we have for finding $\varphi_c(t)$

$$d\varphi_c = \rho_m g_p(\rho_m) d\rho_m/\rho_n. \tag{24}$$

The boundary condition for (24): $\varphi_c = \varphi^*$ with $\rho_m = \rho_1$ [$\varphi^* = G_c(\rho_n)$ with $\rho_m = \rho_1$].

Whence $\varphi_c = \int_{\rho_1}^{\rho_m} \frac{\rho_m g_p(\rho_m)}{\rho_n}\, d\rho_m + \varphi^*$.

The dependence of ρ_n on ρ_m is found by means of the equation

$$\int_{\rho_n^*}^{\rho_n} \rho_n g_c(\rho_n)\, d\rho_n = \int_{\rho_1}^{\rho_m} \rho_m g_p(\rho_m)\, d\rho_m, \tag{25}$$

and ρ_n^* is found from (20) with $\rho_m = \rho_1$. Integration of (25) gives

$$\frac{1}{f} \ln\left[\frac{(3\rho_n - 1)^{7/3}(1-\rho_n^*)}{(3\rho_n^* - 1)^{7/3}(1-\rho_n)}\right] = \frac{(\rho_m - \rho_1)}{(1-\rho_m)(1-\rho_1)} + 2\ln\left(\frac{1-\rho_1}{1-\rho_m}\right). \tag{26}$$

Now by calculating $\varphi_c(t)$ it is possible to find all of the integration functions and consequently the distribution of density and pressing force. Results of the calculation are presented in Fig. 2 (curves 2 and 3).

<u>The Case when a Prandtl Friction Rule Operates over the Whole Length of the Seat of Deformation.</u> The start of this period is determined by the condition $\rho_n = \rho_1$. From (26) density under the piston ρ_m' at the start of this period is found. The distribution of density and velocity is described by relationships (21) and (22). Boundary conditions: $v_z = 0$ with $z = 0$, $v_z = \dot{h}$ with $z = h$. By carrying out transformation similar to previously we have

$$\varphi_p = \int_{\rho_m'}^{\rho_m} \frac{\rho_m g_p(\rho_m)}{\rho_n} d\rho_m + \varphi'. \tag{27}$$

By using the equation $\varphi_p(t) = \varphi_c(t) - G_c(\rho_1) + G_p(\rho_1)$ we write $\varphi' = \varphi_p(\rho_m')$. The density at the bottom of the mold is obtained from

$$\int_{\rho_m'}^{\rho_m} \rho_m g_p(\rho_m)\, d\rho_m = \int_{\rho_1}^{\rho_n} \rho_n g_p(\rho_n)\, d\rho_n. \tag{28}$$

Results of calculating the distribution of density from relationships (27) and (28) are presented in Fig. 2 (curves 1).

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